Products: Signal generators, spectrum analyzers, test receivers, network analyzers, power meters, audio analyzers

dB or not dB?

Everything you ever wanted to know about decibels but were afraid to ask…

Application Note 1MA98

True or false: 30 dBm + 30 dBm = 60 dBm? Why does 1% work out to be -40 dB one time but then 0.1 dB or 0.05 dB the next time? These questions sometimes leave even experienced engineers scratching their heads. Decibels are found everywhere, including power levels, voltages, reflection coefficients, noise figures, field strengths and more. What is a decibel and how should we use it in our calculations? This Application Note is intended as a refresher on the subject of decibels.

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Content

1 Introduction

%, dB, dBm and dB(µV/m) are important concepts that every engineer should understand in his (or her) sleep. Because if he doesn't, he is bound to be at a disadvantage in his work. When these terms come up in discussions with customers or colleagues, he will have trouble focusing on the real issue if he is busy wondering whether 3 dB means a factor of 2 or 4 (or something else). It is well worth the effort to review these concepts from time to time and keep familiar with them.

While this Application Note is not intended as a textbook, it will help to refresh your knowledge of this topic if you studied it before or provide a decent introduction if it is new to you.

When it comes to writing formulas and units, we have followed the international standards specified in ISO 31 and IEC 27 (or else we have indicated where it is common practice to deviate from the standard).

2 Why use decibels in our calculations?

Engineers have to deal with numbers on an everyday basis, and some of these numbers can be very large or very small. In most cases, what is most important is the ratio of two quantities. For example, a mobile radio base station might transmit approx. 80 W of power (antenna gain included). The mobile phone receives only about 0.000 000 002 W, which is 0.000 000 002 5% of the transmitted power.

Whenever we must deal with large numerical ranges, it is convenient to use the logarithm of the numbers. For example, the base station in our example transmits at +49 dBm while the mobile phone receives -57 dBm, producing a power difference of +49 dBm - (-57 dBm) = 106 dB.

Another example: If we cascade two amplifiers with power gains of 12 and 16, respectively, we obtain a total gain of 12 times 16 = 192 (which you can hopefully calculate in your head – do you?). In logarithmic terms, the two amplifiers have gains of 10.8 dB and 12 dB, respectively, producing a total gain of 22.8 dB, which is definitely easier to calculate.

When expressed in decibels, we can see that the values are a lot easier to manipulate. It is a lot easier to add and subtract decibel values in your head than it is to multiply or divide linear values. This is the main reason we like to make our computations in decibels.

3 Definition of dB

Although the base 10 logarithm of the ratio of two power levels is a dimensionless quantity, it has units of "Bel" in honor of the inventor of the telephone (Alexander Graham Bell). In order to obtain more manageable numbers, we use the dB (**deci**bel, where "deci" stands for one tenth) instead of the Bel for computation purposes. We have to multiply the Bel values by 10 (just as we need to multiply a distance by 1000 if we want to use millimeters instead of meters).

$$a = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \text{dB}$$

As mentioned above, the advantage of using decibels is that the huge range of the signals commonly encountered in telecommunications and radio frequency engineering can be represented with more manageable numbers.

Example: *P_1 is equal to 200 W and P_2 is equal to 100 mW. What is their ratio a in dB?*

$$a = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right)\mathrm{dB} = 10 \cdot \log_{10}(2000)\,\mathrm{dB} = 33.01\,\mathrm{dB}$$

Of course, before dividing these power levels, we have to convert them to the same unit, i.e. W or mW. We won't obtain the correct result if we just divide 200 by 100.

Nowadays, we use base 10 logarithms almost exclusively. The abbreviation for a base 10 logarithm is **lg**. In older textbooks, you will sometimes see the natural logarithm used, which is the base e logarithm (e = approx. 2.718). In this Application Note, we use only the base 10 logarithm which we abbreviate with lg without indicating the base furtheron.

Of course, it is also possible to convert decibels back to linear values. We must first convert from dB to Bel by dividing the value by 10. Then, we must raise the number 10 (since we are using a base 10 logarithm) to this power:

$$\frac{P_1}{P_2} = 10^{\frac{a/\mathrm{dB}}{10}}$$

Example: *a = 23 dB, what is P1 / P2?*
After first computing 23 / 10 = 2.3, we obtain:

$$\frac{P_1}{P_2} = 10^{2.3} = 199.5$$

4 What does dBm mean?

If we refer an arbitrary power level to a fixed reference quantity, we obtain an absolute quantity from the logarithmic power ratio.

The reference quantity most commonly used in telecommunications and radio frequency engineering is a power of 1 mW (one thousandth of a Watt) into 50 Ohm. This reference quantity is designated by appending an m (for mW) to dB to give dBm.

The general power ratio P_1 to P_2 now becomes a ratio of P_1 to 1 mW, indicated in dBm.

$$P = 10 \cdot \lg\left(\frac{P_1}{1\,mW}\right)\mathrm{dBm}$$

In accordance with the IEC 27 standard, however, there is a different way of writing this formula. Levels are to be indicated with *L* and the reference value must be indicated explicitly. This turns our formula into:

$$L_{P(\mathrm{re\,1\,mW})} = 10 \cdot \lg\left(\frac{P_1}{1\,mW}\right)\mathrm{dB}$$

or the short form:

$$L_{P/1\,\mathrm{mW}} = 10 \cdot \lg\left(\frac{P_1}{1\,mW}\right)\mathrm{dB}$$

We would now write $L_{P/1mW}$ = 7 dB, for example. According to IEC 27, we are prohibited from using the expression 7 dBm. Of course, it is much more common to write dBm. So we will continue using dBm throughout this paper.

To give you a feeling for the orders of magnitude which tend to occur, here are some examples: The output power range of signal generators extends typically from -140 dBm to +20 dBm or 0.01 fW (femto Watt) to 0.1 W. Mobile radio base stations transmit at +43 dBm or 20 W. Mobile phones transmit at +10 dBm to +33 dBm or 10 mW to 2 W. Broadcast transmitters operate at +70 dBm to +90 dBm or 10 kW to 1 MW.

5 What's the difference between voltage decibels and power decibels?

First of all, please forget everything you've ever heard about voltage and power decibels. There is only one type of decibel, and it represents a ratio of two power levels P_1 and P_2. Of course, any power level can be expressed as a voltage if we know the resistance.

$$P_1 = \frac{U_1^{\,2}}{R_1} \text{ and } P_2 = \frac{U_2^{\,2}}{R_2}$$

We can compute the logarithmic ratio as follows:

$$a = 10 \cdot \lg\left(\frac{P_1}{P_2}\right)\mathrm{dB} = 10 \cdot \lg\left(\frac{U_1^{\,2}}{U_2^{\,2}} \cdot \frac{R_2}{R_1}\right)\mathrm{dB}$$

Using the following 3 familiar identities,

$$\log\left(\frac{1}{x}\right) = -\log(x)$$

$$\log\left(x^y\right) = y \cdot \log(x)$$

$$\log(xy) = \log(x) + \log(y)$$

we obtain (again using lg to mean the base 10 logarithm):

$$a = 10 \cdot \lg\left(\frac{P_1}{P_2}\right)\mathrm{dB} = 10 \cdot \lg\left(\frac{U_1^{\,2}}{U_2^{\,2}} \cdot \frac{R_2}{R_1}\right)\mathrm{dB} = 20 \cdot \lg\left(\frac{U_1}{U_2}\right)\mathrm{dB} - 10 \cdot \lg\left(\frac{R_1}{R_2}\right)\mathrm{dB}$$

Note the minus sign in front of the resistance term.

In most cases, the reference resistance is equal for both power levels, i.e. $R_1 = R_2$. Since

$$10 \cdot \lg(1) = 0$$

we can simplify as follows:

$$a = 10 \cdot \lg\left(\frac{P_1}{P_2}\right)\text{dB} = 20 \cdot \lg\left(\frac{U_1}{U_2}\right)\text{dB}$$ **(simplified for $R_1 = R_2$!)**

This also explains why we use 10·lg for power ratios and 20·lg for voltage ratios.

Caution: *(Very important!) This formula is valid only if R1 = R2. If, as sometimes occurs in television engineering, we need to take into account a conversion from 75 Ohm to 50 Ohm, we need to consider the ratio of the resistances.*

Conversion back to linear values is the same as before. For voltage ratios, we must divide the value *a* by 20 since we use U^2 and **deci**bels (20 = 2·10, 2 from U^2, 10 from deci).

$$\frac{P_1}{P_2} = 10^{\frac{a/\text{dB}}{10}}$$

$$\frac{U_1}{U_2} = 10^{\frac{a/\text{dB}}{20}}$$

6 What is a level?

As we saw above, dBm involves a reference to a power level of 1 mW. Other frequently used reference quantities include 1 W, 1 V, 1 µV and also 1 A or 1 µA. They are designated as dB (W), dB (V), dB (µV), dB (A) and dB (µA), respectively, or in field strength measurements, dB (W/m^2), dB (V/m), dB (µV/m), dB (A/m) and dB (µA/m). As was the case for dBm, the conventional way of writing these units dBW, dBV, dBµV, dBA, dBµA, dBW/m^2, dBV/m, dBµV/m, dBA/m and dBµA/m deviates from the standard, but will be used in this paper.

From the relative values for power level P_1 (voltage U_1) referred to power level P_2 (voltage U_2), we obtain absolute values using the reference values above.

These absolute values are also known as **levels**. A level of 10 dBm means a value which is 10 dB above 1 mW, and a level of -17 dB(µV) means a value which is 17 dB below 1 µV.

When computing these quantities, it is important to keep in mind whether they are power quantities or voltage quantities.

Some examples of power quantities include power, energy, resistance, noise figure and power flux density.

Voltage quantities (also known as field quantities) include voltage, current, electric field strength, magnetic field strength and reflection coefficient.

Examples: *A power flux density of 5 W/m² has the following level:*

$$P = 10 \cdot \lg\left(\frac{5\,\mathrm{W/m^2}}{1\,\mathrm{W/m^2}}\right) = 7\,\mathrm{dB\,(W/m^2)}$$

A voltage of 7 µV can also be expressed as a level in dB(µV):

$$U = 20 \cdot \lg\left(\frac{7\,\mu\mathrm{V}}{1\,\mu\mathrm{V}}\right) = 16.9\,\mathrm{dB\,(\mu V)}$$

Conversion from levels to linear values requires the following formulas:

$$P = 10^{\frac{a/\mathrm{dB}}{10}} \cdot P_{ref}$$

or

$$U = 10^{\frac{u/\mathrm{dB}}{20}} \cdot U_{ref}$$

Examples: *A power level of -3 dB(W) has the following power:*

$$P = 10^{\frac{-3}{10}} \cdot 1\,\mathrm{W} = 0.5 \cdot 1\,\mathrm{W} = 500\,\mathrm{mW}$$

A voltage level of 120 dB(µV) has a voltage of:

$$U = 10^{\frac{120}{20}} \cdot 1\,\mu\mathrm{V} = 1000000 \cdot 1\,\mu\mathrm{V} = 1\,\mathrm{V}$$

7 Attenuation and gain

The linear transfer function a_{lin} of a two-port circuit represents the ratio of the output power to the input power:

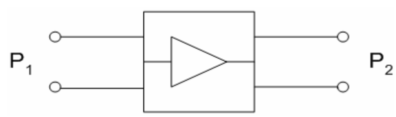

Fig 1: Two-port circuit

$$a_{lin} = \frac{P_2}{P_1}$$

The transfer function is normally specified in dB:

$$a = 10 \cdot \lg \frac{P_2}{P_1} \, \text{dB}$$

If the output power P_2 of a two-port circuit is greater than the input power P_1, then the logarithmic ratio of P_2 to P_1 is positive. This is known as **amplification** or **gain**.

If the output power P_2 of a two-port circuit is less than the input power P_1, then the logarithmic ratio of P_2 to P_1 is negative. This is known as **attenuation** or **loss** (the minus sign is omitted).

Computation of the power ratio or the voltage ratio from the decibel value uses the following formulas:

$$\frac{P_2}{P_1} = 10^{\frac{a/\text{dB}}{10}}$$

or

$$\frac{U_2}{U_1} = 10^{\frac{a/\text{dB}}{20}} \qquad \text{(for } R_{out} = R_{in}\text{)}$$

Conventional amplifiers realize gains of up to 40 dB in a single stage, which corresponds to voltage ratios up to 100 and power ratios up to 10000. With higher values, there is a risk of oscillation in the amplifier. However, higher gain can be obtained by connecting multiple stages in series. The oscillation problem can be avoided through suitable shielding.

The most common attenuators have values of 3 dB, 6 dB, 10 dB and 20 dB. This corresponds to voltage ratios of 0.7, 0.5, 0.3 and 0.1 or power ratios of 0.5, 0.25, 0.1 and 0.01. Here too, we must cascade multiple attenuators to obtain higher values. If we attempt to obtain higher attenuation in a single stage, there is a risk of crosstalk.

Series connection of two-port circuits:

In the case of series connection (cascading) of two-port circuits, we can easily compute the total gain (or total attenuation) by adding the decibel values.

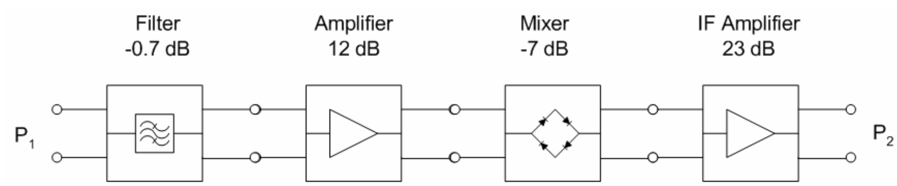

Fig 2: Cascading two-port circuits

The total gain is computed as follows:

$$a = a_1 + a_2 + ... + a_n$$

Example: *Fig. 2 shows the input stages of a receiver. The total gain* a *is computed as follows:*

a = -0.7 dB + 12 dB - 7 dB + 23 dB = 27.3 dB.

8 Conversion from decibels to percentage and vice versa

The term "percent" comes from the Latin and literally means "per hundred". 1% means one hundredth of a value.

$$1\,\%\text{ of }x = 0{,}01 \cdot \text{x}$$

When using percentages, we need to ask two questions:

- Are we calculating voltage quantities or power quantities?
- Are we interested in x% of a quantity or x% more or less of a quantity?

As mentioned above, voltage quantities are voltage, current, field strength and reflection coefficient, for example.

Power quantities include power, resistance, noise figure and power flux density.

Converting % voltage to decibels and vice versa

x% of a voltage quantity is converted to decibels as follows:

$$a = 20 \cdot \lg \frac{x}{100}\,\text{dB}$$

In other words: To obtain a value of x% in decibels, we must first convert the percentage value x to a rational number by dividing x by 100. To convert to decibels, we multiply the logarithm of this rational number by 20 (voltage quantity: 20) as shown above.

Example: *Assume the output voltage of a two-port circuit is equal to 3% of the input voltage. What is the attenuation* a *in dB?*

$$a = 20 \cdot \lg \frac{3}{100}\,\text{dB} = -30.46\,\text{dB}$$

We can convert a decibel value *a* to a percentage as follows:

$$x = 100\,\% \cdot 10^{\frac{a/\text{dB}}{20}}$$

Example: *Calculate the output voltage of a 3 dB attenuator as a percentage of the input voltage.*

$$x = 100\,\% \cdot 10^{\frac{-3}{20}} = 70.8\,\%$$

The output voltage of a 3 dB attenuator is equal to 71% of the input voltage.
Note: Attenuation means negative decibel values!

Converting % power to decibels and vice versa

x% of a power quantity is converted to decibels as follows:

$$a = 10 \cdot \lg \frac{x}{100}\,\text{dB}$$

To obtain a value in decibels, we first convert the percentage value x to a rational number (as shown above) by dividing the number by 100. To convert to decibels (as described in section 2), we multiply the logarithm of this rational number by 10 (power quantity: 10).

Example: *Assume the output power of a two-port circuit is equal to 3% of the input power. What is the attenuation* a *in dB?*

3 %·P = 0.03·P

$$a = 10 \cdot \lg \frac{3}{100}\,\text{dB} = -15.23\,\text{dB}$$

We can convert a decibel value *a* to a percentage as follows:

$$x = 100\,\% \cdot 10^{\frac{a/\text{dB}}{10}}$$

Example: *Calculate the output power of a 3 dB attenuator as a percentage of the input power.*

$$x = 100\,\% \cdot 10^{\frac{-3}{10}} = 50.1\,\%$$

The power at the output of a 3 dB attenuator is half as large (50%) as the input power.
Note: As above, attenuation means negative decibel values!

Converting % voltage more or less to decibels

x% more (or less) of a value means that we add (or subtract) the given percentage to (or from) the starting value. For example, if the output voltage U_2 of an amplifier is supposed to be x% greater than the input voltage U_1, we calculate as follows:

$$U_2 = U_1 + x\% \cdot U_1 = U_1\left(1 + \frac{x}{100}\right)$$

If the output voltage is less than the input voltage, then x should be a negative value.

Conversion to a decibel value requires the following formula:

$$a = 20 \cdot \lg\left(1 + \frac{x}{100}\right)\text{dB}$$

Note: Use a factor of 20 for voltage quantities.

Example: *The output voltage of an amplifier is 12.2% greater than the input voltage. What is the gain in decibels?*

$$a = 20 \cdot \lg\left(1 + \frac{12.2}{100}\right)\text{dB} = 1\,\text{dB}$$

Note that starting with even relatively small percentage values, a given plus percentage will result in a different decibel value than its corresponding minus percentage.

20% more results in	+1.58 dB
20% less results in	-1.94 dB

Converting % power more or less to decibels

Analogous to the voltage formula, we have the following for power:

$$P_2 = P_1 + x\% \cdot P_1 = P_1\left(1 + \frac{x}{100}\right)$$

Conversion to a decibel value requires the following formula:

$$a = 10 \cdot \lg\left(1 + \frac{x}{100}\right)\text{dB}$$

Note: Use a factor of 10 for power quantities.

Example: *The output power of an attenuator is 20% less than the input power. What is the attenuation in decibels?*

$$a = 10 \cdot \lg\left(1 + \frac{-20}{100}\right)\text{dB} = -0.97\,\text{dB} \approx -1\,\text{dB}$$

As before, we can expect asymmetry in the decibel values starting with even small percentage values.

9 Using dB values in computations

This section demonstrates how to add power levels and voltages in logarithmic form, i.e. in decibels.

Adding power levels

30 dBm + 30 dBm = 60 dBm? Of course not! If we convert these power levels to linear values, it is obvious that 1 W + 1 W = 2 W. This is 33 dBm and not 60 dBm. But this is true only if the power levels to be added are uncorrelated. Uncorrelated means that the instantaneous values of the power levels do not have a fixed phase relationship with one another.

Note: *Power levels in logarithmic units need to be converted prior to addition so that we can add linear values. If it is more practical to work with decibel values after the addition, we have to convert the sum back to dBm.*

Example: *We want to add three signals P1, P2 and P3 with levels of 0 dBm, +3 dBm and -6 dBm. What is the total power?*

$$P_1 = 10^{\frac{0}{10}} = 1\,\mathrm{mW}$$

$$P_2 = 10^{\frac{3}{10}} = 2\,\mathrm{mW}$$

$$P_3 = 10^{\frac{-6}{10}} = 0.25\,\mathrm{mW}$$

$$P = P_1 + P_2 + P_3 = 3.25\,\mathrm{mW}$$

Converting back to decibels we get

$$P = 10 \cdot \lg\left(\frac{3.25\,\mathrm{mW}}{1\,\mathrm{mW}}\right)\mathrm{dBm} = 5.12\,\mathrm{dBm}$$

The total power is 5.12 dBm.

Measuring signals at the noise limit

One common task involves measurement of weak signals close to the noise limit of a test instrument such as a receiver or a spectrum analyzer. The test instrument displays the sum total of the inherent noise and signal power, but it should ideally display only the signal power. The prerequisite for the following calculation is that the test instrument must display the RMS power of the signals. This is almost always the case with power meters, but with spectrum analyzers it is necessary to switch on the RMS detector.

First, we determine the inherent noise P_r of the test instrument by turning off the signal. Then, we measure the signal with noise P_{tot}. We can obtain the power P of the signal alone by subtracting the linear power values.

Example: *The displayed noise P_r of a power meter is equal to -70 dBm. When a signal is applied, the displayed value increases to P_{tot}*

= -65 dBm. What is the power of the signal P in dBm?

$$P_r = 10^{\frac{-70}{10}}\ \text{mW} = 0.000\,000\,1\,\text{mW}$$

$$P_{tot} = 10^{\frac{-65}{10}}\ \text{mW} = 0.000\,000\,316\,\text{mW}$$

$$P = P_{tot} - P_r$$

$$P = 0.000\,000\,316\,\text{mW} - 0.000\,000\,1\,\text{mW} = 0.000\,000\,216\,\text{mW}$$

$$P = 10 \cdot \lg \frac{0.000\,000\,216\,\text{mW}}{1\,\text{mW}}\,\text{dBm} = -66.6\,\text{dBm}$$

The signal power P is -66.6 dBm.

We can see that without any compensation, the noise of the test instrument will cause a display error of 1.6 dB, which is relatively large for a precision test instrument.

Adding voltages

Likewise, we can add decibel values for voltage quantities only if we convert them from logarithmic units beforehand. We must also know if the voltages are correlated or uncorrelated. If the voltages are correlated, we must also know the phase relationship of the voltages.

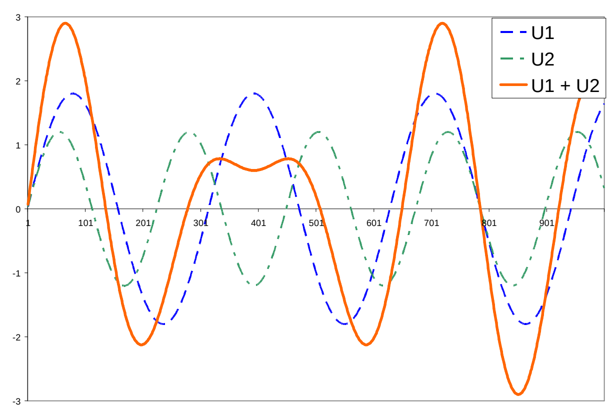

Fig 3: Addition of two uncorrelated voltages

We add uncorrelated voltages quadratically, i.e. we actually add the associated power levels. Since the resistance to which the voltages are applied is the same for all of the signals, the resistance will disappear from the formula:

$$U = \sqrt{U_1^{\,2} + U_2^{\,2} + \ldots + U_n^{\,2}}$$

If the individual voltages are specified as levels, e.g. in dB(V), we must first convert them to linear values.

Example: We add three uncorrelated voltages U_1 = 0 dB(V), U_2 = -6 dB(V) and U_3 = +3 dB(V) as follows to obtain the total voltage U:

$$U_1 = 10^{\frac{U_1/\text{dB(V)}}{20}} \cdot U_{ref} = 10^{\frac{0}{20}} \cdot 1\,\text{V} = 1\,\text{V}$$

$$U_2 = 10^{\frac{U_2/\text{dB(V)}}{20}} \cdot U_{ref} = 10^{\frac{-6}{20}} \cdot 1\,\text{V} = 0.5\,\text{V}$$

$$U_3 = 10^{\frac{U_3/\text{dB(V)}}{20}} \cdot U_{ref} = 10^{\frac{3}{20}} \cdot 1\,\text{V} = 1.41\,\text{V}$$

$$U = \sqrt{U_1{}^2 + U_1{}^2 + U_1{}^2} = \sqrt{1^2 + 0.5^2 + 1.41^2}\ \text{V} = 1.75\,\text{V}$$

After converting U to dB(V), we obtain:

$$U = 20 \log \frac{1.75\,\text{V}}{1\,\text{V}}\,\text{dB(V)} = 4.86\,\text{dB(V)}$$

If the voltages are correlated, the computation becomes significantly more complicated. As we can see from the following figures, the phase angle of the voltages determines the total voltage which is produced.

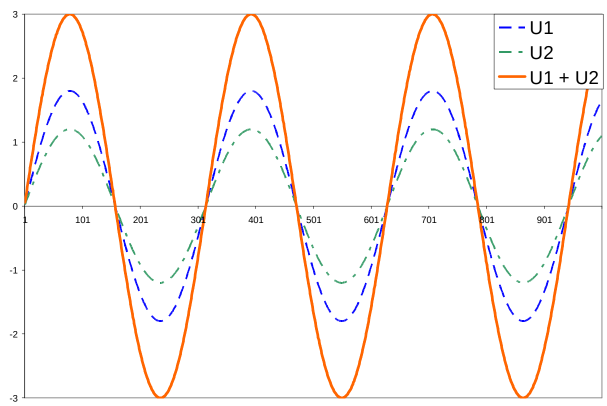

Fig 4: Addition of two correlated voltages, 0° phase angle

Blue represents voltage U_1, green represents voltage U_2 and red represents the total voltage U.

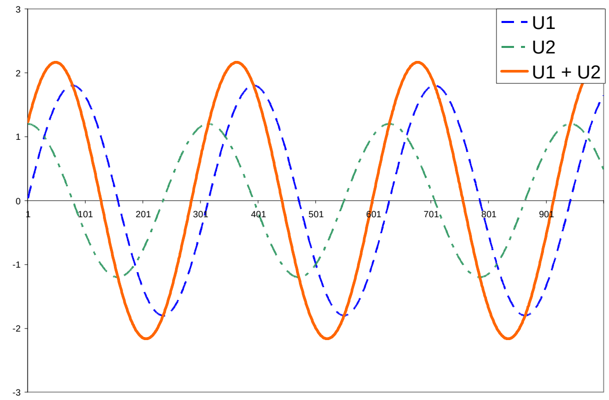

Fig 5: Addition of two correlated voltages, 90° phase angle

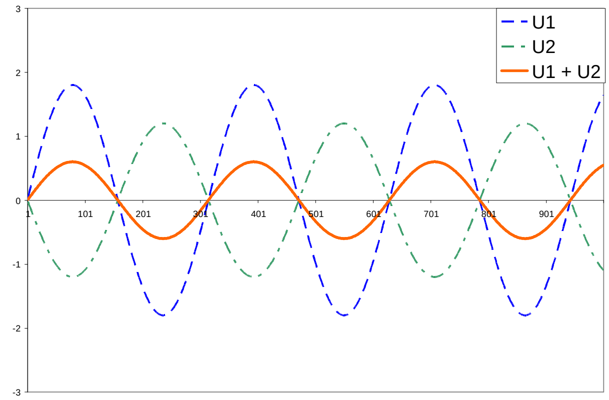

Fig 6: Addition of two correlated voltages, 180° phase angle

The total voltage U ranges from $U_{max} = U_1 + U_2$ for phase angle 0° (in-phase) to $U_{min} = U_1 - U_2$ for phase angle 180° (opposite phase). For phase angles in between, we must form the vector sum of the voltages (see elsewhere for more details).

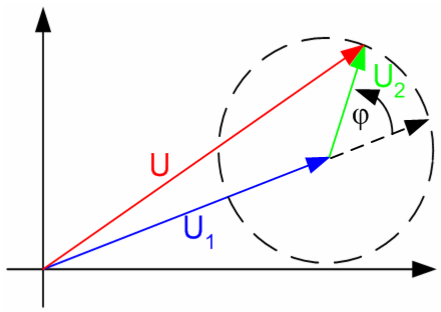

Fig 7: Vector addition of two voltages

In actual practice, we normally only need to know the extreme values of the voltages, i.e. U_{max} and U_{min}.

If the voltages U_1 and U_2 are in the form of level values in dB(V) or dB(µV), we must first convert them to linear values just like we did with uncorrelated voltages. However, the addition is linear instead of quadratic (see the next section about peak voltages).

Peak voltages

If we apply a composite signal consisting of different voltages to the input of an amplifier, receiver or spectrum analyzer, we need to know the peak voltage. If the peak voltage exceeds a certain value, limiting effects will occur which can result in undesired mixing products or poor adjacent channel power. The peak voltage U is equal to:

$$U = U_1 + U_2 + ... + U_n$$

The maximum drive level for amplifiers and analyzers is usually indicated in dBm. In a 50 Ω system, conversion based on the peak voltage is possible with the following formula:

$$P = 20 \cdot \lg \frac{U^2}{50\Omega} \cdot 10^3 \text{ dBm}$$

The factor 10^3 comes from the conversion from Watts to milli Watts

Note that this power level represents the instantaneous peak power and not the RMS value of the power.

10 What do we measure in decibels?

This section summarizes some of the terms and measurement quantities which are typically specified in decibels. This is not an exhaustive list and we suggest you consult the bibliography if you would like more information about this subject. The following sections are structured to be independent of one another so you can consult just the information you need.

Signal-to-noise ratio (S/N)

One of the most important quantities when measuring signals is the signal-to-noise ratio (*S/N*). Measured values will fluctuate more if the S/N degrades. To determine the signal-to-noise ratio, we first measure the signal *S* and then the noise power *N* with the signal switched off or suppressed using a filter. Of course, it is not possible to measure the signal without any noise at all, meaning that we will obtain correct results only if we have a good S/N.

$$SN = \frac{S}{N}$$

Or in dB:

$$SN = 10 \cdot \lg \frac{S}{N} \, \mathrm{dB}$$

Sometimes, distortion is also present in addition to noise. In such cases, it is conventional to determine the signal to noise and distortion (SINAD) as opposed to just the signal-to-noise ratio.

$$SINAD = \frac{S}{N + D}$$

Or in dB:

$$SINAD = 10 \lg \frac{S}{N + D} \, \mathrm{dB}$$

Example: *We would like to measure the S/N ratio for an FM radio receiver. Our signal generator is modulated at 1 kHz with a suitable FM deviation. At the loudspeaker output of the receiver, we measure a signal power level of 100 mW, for example. We now turn off the modulation on the signal generator and measure a noise power of 0.1 µW at the receiver output. The S/N is computed as follows:*

$$SN = 10 \cdot \lg \frac{100 \, \mathrm{mW}}{0.1 \, \mu\mathrm{W}} = 60 \, \mathrm{dB}$$

To determine the SINAD value, we again modulate the signal generator at 1 kHz and measure (as before) a receiver power level of 100 mW. Now, we suppress the 1 kHz signal using a narrow notch filter in the test instrument. At the receiver output, all we now measure is the noise and the harmonic distortion. If the measured value is equal to, say, 0.5 µW, we obtain the SINAD as follows:

$$SINAD = 10 \cdot \lg \frac{100\,\text{mW}}{0{,}5\,\mu\text{W}} = 53\,\text{dB}$$

Noise

Noise is caused by thermal agitation of electrons in electrical conductors. The power *P* which can be consumed by a sink (e.g. receiver input, amplifier input) is dependent on the temperature *T* and also on the measurement bandwidth *B* (please don't confuse bandwidth B with B = Bel!).

$$P = kTB$$

Here, k is Boltzmann's constant 1.38 x 10-23 J K-1 (Joules per Kelvin, 1 Joule = 1 Watt-Second), T is the temperature in K (Kelvin, 0 K corresponds to -273.15°C or –459.67°F) and B is the measurement bandwidth in Hz.

At room temperature (20°C, 68°F), we obtain per Hertz bandwidth a power of:

$$P = kT \cdot 1\,\text{Hz} = 1{,}38 \cdot 10^{-23}\ \text{WsK}^{-1} \cdot 293{,}15\,\text{K} \cdot 1\,\text{Hz} = 4{,}047 \cdot 10^{-21}\ \text{W}$$

If we convert this power level to dBm, we obtain the following:

$$P/\text{Hz} = 10 \cdot \lg\left(\frac{4.047 * 10^{-18}\ \text{mW}}{1\,\text{mW}}\right)\text{dBm} = -173.93\,\text{dBm/Hz}$$

The thermal noise power at a receiver input is equal to -174 dBm per Hertz bandwidth. Note that this power level is not a function of the input impedance, i.e. it is the same for 50 Ω, 60 Ω and 75 Ω systems.

The power level is proportional to bandwidth B. Using the bandwidth factor b in dB, we can compute the total power as follows:

$$b = 10 \cdot \lg\left(\frac{B}{1\,\text{Hz}}\right)\text{dB}$$

$$P = -174\ \text{dBm} + b$$

Example: *An imaginary spectrum analyzer that produces no intrinsic noise is set to a bandwidth of 1 MHz. What noise power will it display?*

$$b = 10 \cdot \lg\left(\frac{1\,\text{MHz}}{1\,\text{Hz}}\right)\text{dB} = 10 \cdot \lg\left(\frac{1000000\ \text{Hz}}{1\,\text{Hz}}\right)\text{dB} = 60\,\text{dB}$$

$$P = -174\,\text{dBm} + 60\,\text{dB} = -114\,\text{dBm}$$

The noise power which is displayed at room temperature at a 1 MHz bandwidth is equal to -114 dBm.

A receiver / spectrum analyzer produces 60 dB more noise with a 1 MHz bandwidth than with a 1 Hz bandwidth. A noise level of -114 dBm is displayed. If we want to measure lower amplitude signals, we need to reduce the bandwidth. However, this is possible only until we reach the bandwidth of the signal. To a certain extent, it is possible to measure signals even if they lie below the noise limit since each additional signal increases the total power which is displayed (see the section on measuring signals at the noise limit above). However, we will quickly reach the resolution limit of the test instrument we are using.

Certain special applications such as deep-space research and astronomy necessitate measurement of very low-amplitude signals from space probes and stars, for example. Here, the only possible solution involves cooling down the receiver input stages to levels close to absolute zero (-273.15°C or –459.67 F).

Averaging noise signals

To display noise signals in a more stable fashion, it is conventional to switch on the averaging function provided in spectrum analyzers. Most spectrum analyzers evaluate signals using what is known as a sample detector and average the logarithmic values displayed on the screen. This results in a systematic measurement error since lower measured values have an excessive influence on the displayed measurement result. The following figure illustrates this effect using the example of a signal with sinusoidal amplitude modulation.

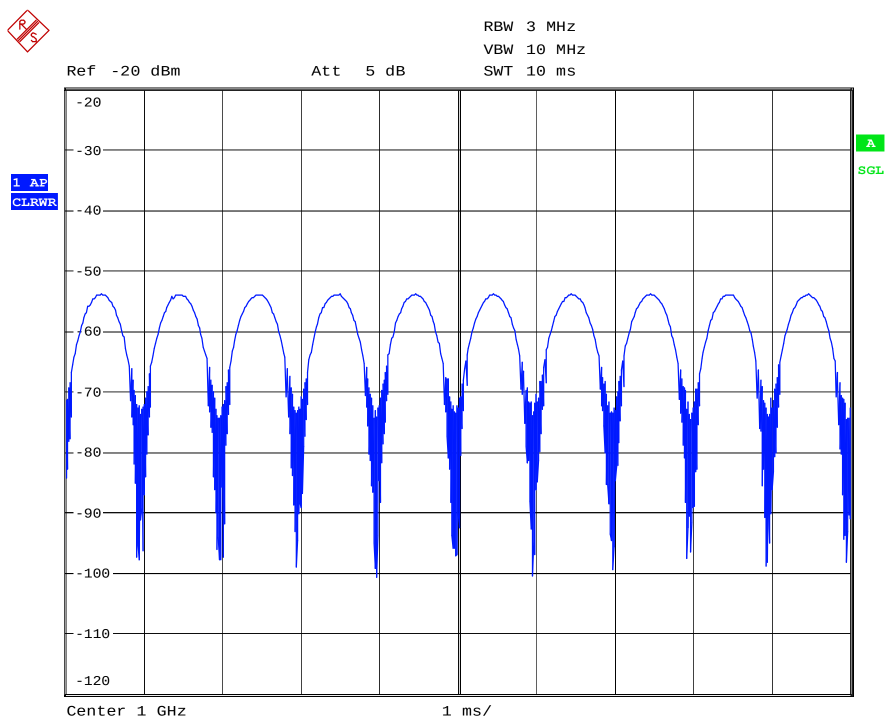

Fig 8: Amplitude-modulated signal with logarithmic amplitude values as a function of time

As we can see here, the sinewave is distorted to produce a sort of heart-shaped curve with an average value which is too low by 2.5 dB. R&S spectrum analyzers use an RMS detector to avoid this measurement error (see [3]).

Noise factor, noise figure

The noise factor F of a two-port circuit is defined as the ratio of the input signal-to-noise ratio SN_{in} to the output signal-to-noise ratio SN_{out}.

$$F = \frac{SN_{in}}{SN_{out}}$$

The signal-to-noise ratio S/N is determined as described above.

If the noise factor is specified in a logarithmic unit, we use the term noise figure (*NF*).

$$NF = 10 \cdot \lg \frac{SN_{in}}{SN_{out}} \text{ dB}$$

When determining the noise figure which results from cascading two-port circuits, it is necessary to consider certain details which are beyond the scope of this Application Note. Details can be found in the relevant technical literature or on the Internet (see [2] and [3]).

Phase noise

An ideal oscillator has an infinitely narrow spectrum. Due to the different physical effects of noise, however, the phase angle of the signal varies slightly which results in a broadening of the spectrum. This is known as **phase noise**.

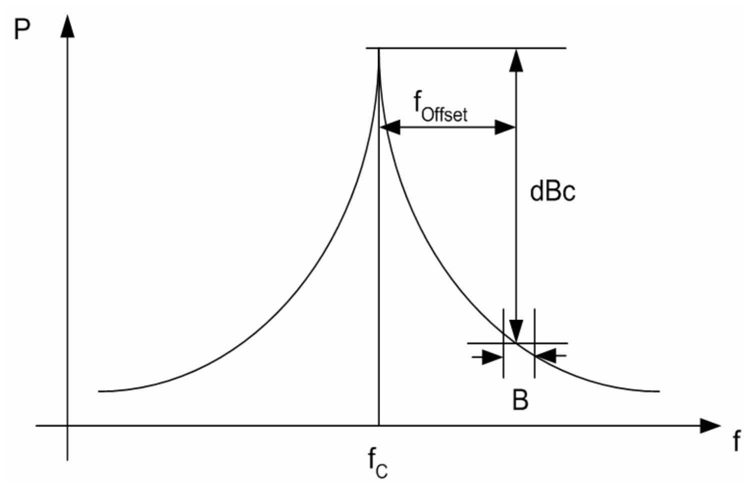

Fig 9: Phase noise of an oscillator

To measure this phase noise, we must determine the noise power of the oscillator P_R as a function of the offset from the carrier frequency f_c (known as the offset frequency f_{Offset}) using a narrowband receiver or a spectrum analyzer in a bandwidth B. We then reduce the measurement bandwidth B computationally to 1 Hz. Now, we reference this power to the power of the carrier P_c to produce a result in dBc (1 Hz bandwidth). The c in dBc stands for “carrier”.

We thus obtain the phase noise, or more precisely, the single sideband (SSB) phase noise *L*:

$$L = 10 \cdot \lg\left(\frac{P_R}{P_c} \cdot \frac{1}{B/1\,\mathrm{Hz}}\right)\mathrm{dBc}$$

dB_c is also a violation of the standard, but it is used everywhere. Conversion to linear power units is possible, but is not conventional.

Data sheets for oscillators, signal generators and spectrum analyzers typically contain a table with phase noise values at different offset frequencies. The values for the upper and lower sidebands are assumed to be equal.

Offset	SSB phase noise
10 Hz	–86 dBc (1 Hz)
100 Hz	–100 dBc (1 Hz)
1 kHz	–116 dBc (1 Hz)
10 kHz	–123 dBc (1 Hz)
100 kHz	–123 dBc (1 Hz)
1 MHz	–144 dBc (1 Hz)
10 MHz	–160 dBc (1 Hz)

Table 1: SSB phase noise at 640 MHz

Most data sheets contain curves for the single sideband phase noise ratio which do not drop off so monotonically as the curve in Fig. 9. This is due to the fact that the phase locked loops (PLLs) used in modern instruments to keep oscillators locked to a reference crystal oscillator result in an improvement but also a degradation of the phase noise as a function of the offset frequency due to certain design problems.

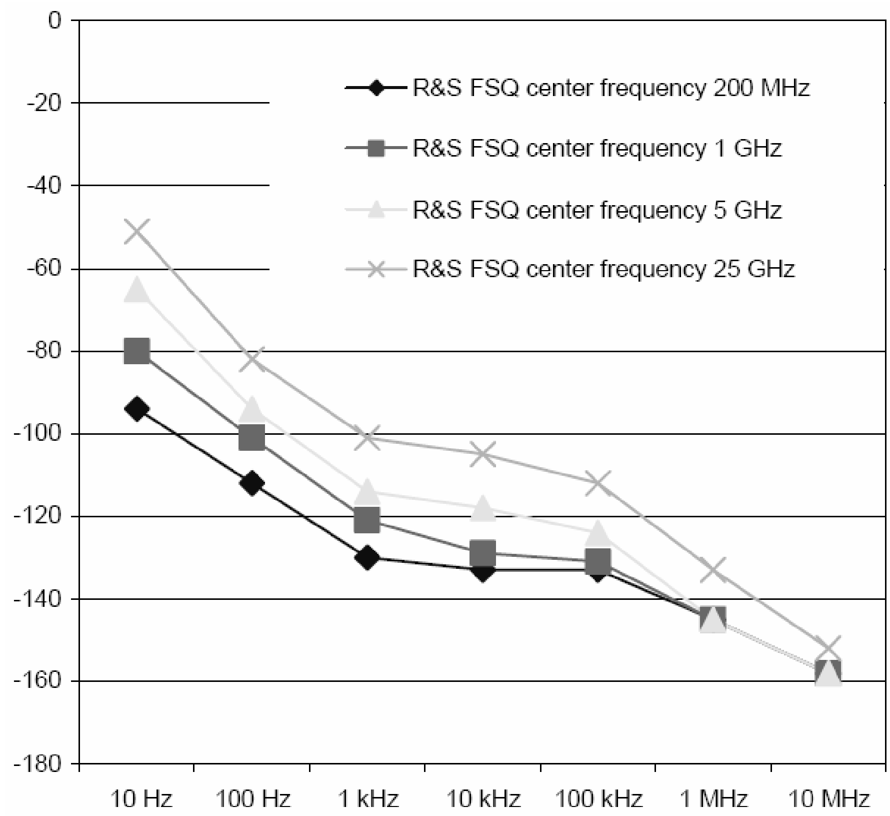

Fig 10: Phase noise curves for the Signal Analyzer R&S®FSQ

When comparing oscillators, it is also necessary to consider the value of the carrier frequency. If we multiply the frequency of an oscillator using a zero-noise multiplier (possible only in theory), the phase noise ratio will degrade proportionally to the voltage, i.e. if we multiply the frequency by 10, the phase noise will increase by 20 dB at the same offset frequency. Accordingly, microwave oscillators are always worse than RF oscillators as a general rule. When mixing two signals, the noise power levels of the two signals add up at each offset frequency.

S parameters

Two-port circuits are characterized by four parameters: S_{11} (input reflection coefficient), S_{21} (forward transmission coefficient), S_{12} (reverse transmission coefficient) and S_{22} (output reflection coefficient).

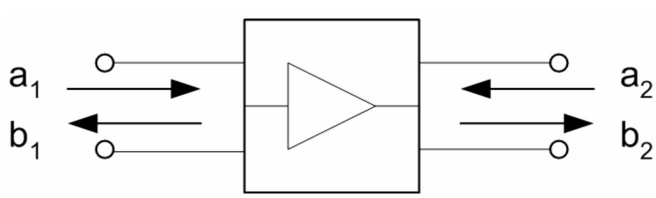

Fig 11: S parameters for a two-port circuit

The S parameters can be computed from the wave quantities a_1, b_1 and a_2, b_2 as follows:

$$S_{11} = \frac{b_1}{a_1} \qquad S_{21} = \frac{b_2}{a_1} \qquad S_{12} = \frac{b_1}{a_2} \qquad S_{22} = \frac{b_2}{a_2}$$

Wave quantities a and b are voltage quantities.

If we have the S parameters in the form of decibel values, the following formulas apply:

$$s_{11} = 20 \cdot \lg S_{11}\ \mathrm{dB} \qquad s_{21} = 20 \cdot \lg S_{21}\ \mathrm{dB}$$

$$s_{12} = 20 \cdot \lg S_{12}\ \mathrm{dB} \qquad s_{22} = 20 \cdot \lg S_{22}\ \mathrm{dB}$$

VSWR and reflection coefficient

Like the reflection coefficient, the voltage standing wave ratio (VSWR) or standing wave ratio (SWR) is a measure of how well a signal source or sink is matched to a reference impedance. VSWR has a range from 1 to infinity and is not specified in decibels. However, the reflection coefficient *r* is.

The relationship between *r* and VSWR is as follows:

$$r = \left| \frac{1 - VSWR}{1 + VSWR} \right|$$

$$VSWR = \left| \frac{1 + r}{1 - r} \right|$$

For VSWR = 1 (very good matching), *r* = 0. For a very high VSWR, *r* approaches 1 (mismatch or total reflection).

r represents the ratio of two voltage quantities. For *r* in decibels, we have a_r:

$$a_r = 20 \cdot \lg\left(\frac{r}{1}\right) \mathrm{dB}$$ (or the other way around:)

$$r = 10^{\frac{a_r / dB}{20}}$$

a_r is called return loss.

For computation of the *VSWR* from the reflection coefficient, *r* is inserted as a linear value.

The following table shows the relationship between VSWR, *r* and *ar*/dB. If you just need a rough approximation of *r* from the VSWR, simply divide the decimal part of the VSWR in half. This works well for VSWR values up to 1.2.

VSWR	r	ar [dB]
1,002	0,001	60
1,004	0,002	54
1,006	0,003	50
1,008	0,004	48
1,01	0,005	46
1,02	0,01	40
1,04	0,02	34
1,1	0,05	26
1,2	0,1	20
1,3	0,13	18
1,4	0,16	15
1,5	0,2	14

Table 2: Conversion from VSWR to reflection coefficient *r* and return loss a_r

Note that for two-port circuits, *r* corresponds to the input reflection coefficient *S11* or the output reflection coefficient *S22*.

Attenuators have the smallest reflection coefficients. Good attenuators have reflection coefficients <5% all the way up to 18 GHz. This corresponds to a return loss of > 26 dB or a VSWR < 1.1. Inputs to test instruments and outputs from signal sources generally have VSWR specifications <1.5, which corresponds to r < 0.2 or r > 14 dB.

Field strength

For field strength measurements, we commonly see the terms power flux density, electric field strength and magnetic field strength.

Power flux density S is measured in W/m^2 or mW/m^2. The corresponding logarithmic units are dB(W/m^2) and dB(mW/m$^{2)}$.

$$S = 10 \cdot \lg\left(\frac{S}{1\,\mathrm{W/m^2}}\right)\mathrm{dB(W/m^2)}$$

$$S = 10 \cdot \lg\left(\frac{S}{1\,\mathrm{mW/m^2}}\right)\mathrm{dBm/m^2}$$

Electric field strength E is measured in V/m or µV/m. The corresponding logarithmic units are dB(V/m) and dB(µV/m).

$$E = 20 \cdot \lg\left(\frac{E/(\mathrm{V/m})}{1/(\mathrm{V/m})}\right)\mathrm{dB(V/m)}$$

$$E = 20 \cdot \lg\left(\frac{E/(\mathrm{\mu V/m})}{1/(\mathrm{\mu V/m})}\right)\mathrm{dB}(\mu\mathrm{V/m})$$

Conversion from dB(V/m) to dB(µV/m) requires the following formula:

$$E\,/\,\mathrm{dB}(\mu\mathrm{V/m}) = E/\mathrm{dB(V/m)} + 120\,\mathrm{dB}$$

Addition of 120 dB corresponds to multiplication by 10^6 in linear units.
1 V = 10^6 µV.

Example: -80 dB(V/m) = -80 dB(µV/m) + 120 dB = 40 dB(µV/m)

Magnetic field strength H is measured in A/m or µA/m. The corresponding logarithmic units are dB(A/m) and dB(µA/m).

$$H = 20 \cdot \lg\left(\frac{H/(\text{A/m})}{1(\text{A/m})}\right) \text{dB(A/m)}$$

$$H = 20 \cdot \lg\left(\frac{H/(\mu\text{A/m})}{1(\mu\text{A/m})}\right) \text{dB}(\mu\text{A/m})$$

Conversion from dB(A/m) to dB(µA/m) requires the following formula:

$$H/dB(\mu A/m) = H/(\text{dB/A}) + 120\,\text{dB}$$

Example: 20 dB(µA/m) = 20 dB(µA/m) – 120 dB = -100 dB(A/m)

For additional information on the topic of field strength, see [1].

Antenna gain

Antennas generally direct electromagnetic radiation into a certain direction. The power gain G which results from this at the receiver is specified in decibels with respect to a reference antenna. The most common reference antennas are the isotropic radiator and the λ/2 dipole. The gain is specified in dB_i or dB_D. If the power gain is needed in linear units, the following formula can be used for conversion:

$$G_{lin} = 10^{\frac{G/dB_i}{10}} \text{ or } G_{lin} = 10^{\frac{G/dB_D}{10}}$$

For more details about antenna gain and the term antenna factor, see [1].

Crest factor

The ratio of the peak power to the average thermal power (RMS value) of a signal is known as the crest factor. A sinusoidal signal has a peak value which is 2 times greater than the RMS value, meaning the crest factor is 2, which equals 3 dB.

For modulated RF signals, the crest factor is referred to the peak value of the modulation envelope instead of the peak value of the RF carrier signal. A frequency-modulated (FM) signal has a constant envelope and thus a crest factor of 1 (0 dB).

If we add up many sinusoidal signals, the peak value can theoretically increase up to the sum of the individual voltages. The peak power P_s would then equal:

$$P_s = \frac{(U_1 + U_2 + \ldots + U_n)^2}{R}$$

The RMS power P is obtained by adding up the individual power levels:

$$P = \frac{U^2}{R} = \frac{U_1{}^2}{R} + \frac{U_2{}^2}{R} + \ldots + \frac{U_n{}^2}{R}$$

We thus obtain a crest factor C_F equal to:

$$C_F = \frac{P_s}{P}$$

$$C_F = 10 \cdot \lg \frac{P_s}{P} \, \mathrm{dB}$$

The more (uncorrelated) signals we add up, the less probable it becomes that the total of the individual voltages will be reached due to the different phase angles. The crest factor fluctuates around a level of about 11 dB. The signal has a noise-like appearance.

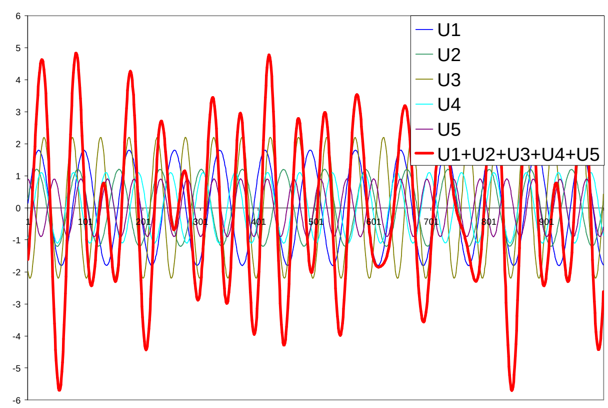

Fig 12: A noise-like signal with a crest factor of 11 dB

Examples: The crest factor of noise is equal to approx. 11 dB. OFDM signals as are used in DAB, DVB-T and WLAN also have crest factors of approx. 11 dB. The CDMA signals stipulated by the CDMA2000 and UMTS mobile radio standards have crest factors ranging up to 15 dB, but they can be reduced to 7 dB to 9 dB using special techniques involving the modulation data. Except for bursts, GSM signals have a constant envelope due to the MSK modulation and thus a crest factor of 0 dB. EDGE signals have a crest factor of 3.2 dB due to the filter function of the 8PSK modulation (also excluding bursts).

Fig 13 shows the so called Complementary Cumulative Distribution Function (CCDF) of a noise like signal. The Crest Factor is that point of the measurement curve, where it reaches the x-axis. In the picture this is at appr. 10.5 dB.

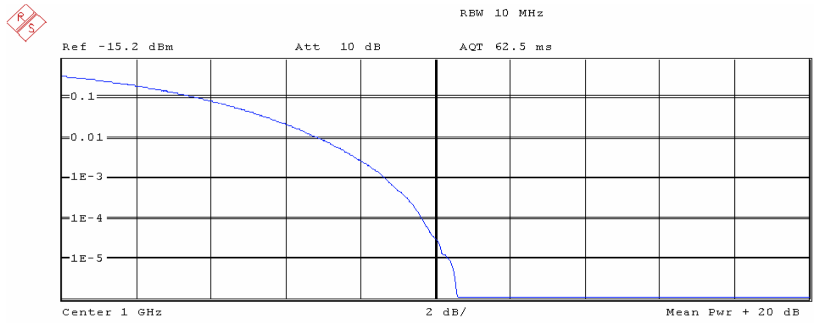

Fig 13: Crest factor measured with the Signal Analyzer R&S®FSQ

Channel power and adjacent channel power

Modern communications systems such as GSM, CDMA2000 and UMTS manage a huge volume of calls. To avoid potential disruptions and the associated loss of revenue, it is important to make sure that exactly the permissible channel power level P_{ch} (where ch stands for channel) is available in the useful channel and no more. The power in the useful channel is most commonly indicated as the level L_{ch} in dBm.

$$L_{ch} = 10 \cdot \lg\left(\frac{P_{ch}}{1\,\mathrm{mW}}\right)\mathrm{dBm}$$

This is normally 20 W or 43 dBm.

In the adjacent channels, the power may not exceed the value P_{adj}. This value is measured as a ratio to the power in the useful channel L_{ACPR} (ACPR = adjacent channel power ratio) and is specified in dB.

$$L_{ACPR} = 10 \cdot \lg\left(\frac{P_{adj}}{P_{ch}}\right)\mathrm{dB}$$

Here, values of -40 dB (for mobile radio devices) down to -70 dB (for UMTS base stations) are required in the immediately adjacent channel and correspondingly higher values in the alternate channels.

When measuring the power levels, it is important to consider the bandwidth of the channels. It can be different for the useful channel and the adjacent channel. Example (CDMA2000): Useful channel 1.2288 MHz, adjacent channel 30 kHz. Sometimes, it is also necessary to select a particular type of modulation filtering, e.g. square-root-cosine-roll-off.

Modern spectrum analyzers have built-in measurement functions which automatically take into account the bandwidth of the useful channel and adjacent channel as well as the filtering. For more information, see also [3].

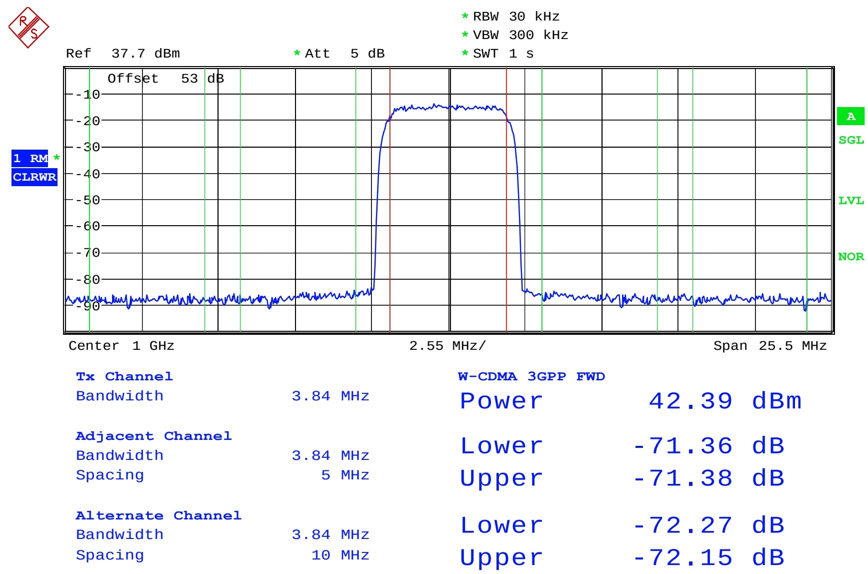

Fig 14: Adjacent channel power for a UMTS signal, measured with the Signal Analyzer R&S®FSQ

Modulation quality EVM

Ideally, we would like to be able to decode signals from digitally modulated transmitters with as few errors as possible in the receiver. Over the course of the transmission path, noise and interference are superimposed in an unavoidable process. This makes it all the more important for the signal from the transmitter to exhibit good quality. One measure of this quality is the deviation from the ideal constellation point. The figure below illustrates this based on the example of QPSK modulation.

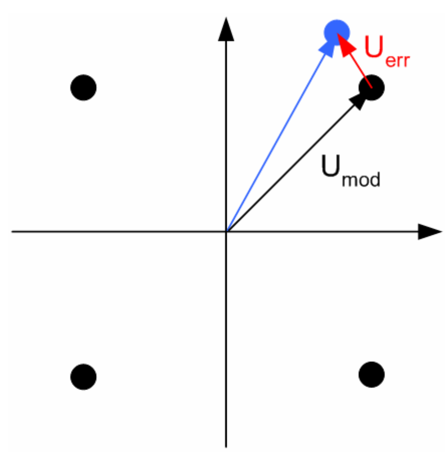

Fig 15: Modulation error

To determine the modulation quality, the magnitude of the error vector U_{err} is referenced to the nominal value of the modulation vector U_{mod}. This quotient is known as the vector error or the error vector magnitude (EVM) and is specified as a percentage or in decibels.

$$EVM_{lin} = \frac{|U_{err}|}{|U_{\mathrm{mod}}|} \cdot 100\,\%$$

$$EVM = 20 \cdot \lg\left(\frac{|U_{err}|}{|U_{\mathrm{mod}}|}\right) \mathrm{dB}$$

We distinguish between the peak value EVM_{peak} occurring over a certain time interval and the RMS value of the error EVM_{RMS}.

Note that these vectors are voltages. This means we must use 20·lg in our calculations. An EVM of 0.3% thus corresponds to -50 dB.

Dynamic range of A/D and D/A converters

Important properties of analog to digital (A/D) and digital to analog (D/A) converters include the clock frequency f_{clock} and the number of data bits *n*. For each bit, we can represent twice (or half, depending on our point of view) the voltage. We thus obtain a dynamic range *D* of 6 dB per bit (as we have already seen, 6 dB corresponds to a factor of 2 for a voltage quantity). There is also a system gain of 1.76 dB for measurement of sine shaped signals.

$$D = 20 \cdot \lg(2^n) + 1.76\,\mathrm{dB}$$

Example: A 16-bit D/A converter has a dynamic range of 96.3 dB + 1.76 dB = 98 dB.

In practice, A/D and D/A converters exhibit certain nonlinearities which make it impossible to achieve their full theoretical values. In addition, clock jitter and dynamic effects mean that converters have a reduced dynamic range particularly at high clock frequencies. A converter is then specified using what is known as the spurious-free dynamic range or the number of effective bits.

Example: An 8-bit A/D converter is specified as having 6.3 effective bits at a clock frequency of 1 GHz. It thus produces a dynamic range of 37.9 dB +1.76 dB = 40 dB.

For a 1 GHz clock frequency, an A/D converter can handle signals up to 500 MHz (Nyquist frequency). If we use only a fraction of this bandwidth, we can actually gain dynamic range by using decimation filters. For example, an 8-bit converter can achieve 60 dB or more dynamic range instead of only 50 dB (= 8 · 6 + 1.76 dB).

Based on the dynamic range, we can compute the number of effective bits as follows:

$$2^n = 10^{\frac{D/dB - 1.76}{20}}$$

With $n = \log_2(2^n)$ ($\log_2$ is the base 2 logarithm) and

$$\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)} \text{ or } \log_{10}(10^x) = x$$

we obtain:

$$n/Bit = \frac{\log_{10}\left(10^{\frac{D/dB-1.76}{20}}\right)}{\log_{10}(2)} = \frac{\frac{D/dB-1.76}{20}}{\log_{10}(2)} = \frac{D/dB-1.76}{20\log_{10}(2)}$$

Example: *How many effective bits does an A/D converter have with a dynamic range of 70 dB?*
We compute as follows:
70 dB – 1.76 dB = 68.2 dB and 20log$_{10}$(2) = 6.02
68.2 / 6,02 = 11,3

We thus obtain a result of 11.3 effective bits.

dB (FS) (Full Scale)

Analog to digital converters and digital to analog converters have a maximum dynamic range which is determined by the range of numbers they can process. For example, an 8-bit A/D converter can handle numbers from 0 to a maximum of 2^8 - 1= 255. This number is also known as the full-scale value (n_{FS}). We can specify the drive level n of such converters with respect to this full-scale value and represent this ratio logarithmically.

$$a = 20 \cdot \lg\left(\frac{n}{n_{FS}}\right) \mathrm{dB(FS)}$$

Example: *A 16-bit A/D converter has a range of values from 0 to 2^{16} - 1 = 65535. If we drive this converter with the voltage which is represented by a numerical value of 32767, we have:*

$$a = 20 \cdot \lg\left(\frac{32737}{65535}\right) \mathrm{dB(FS)} = \text{-6,02}\ \mathrm{dB(FS)}$$

If the converter is expected to represent positive and negative voltages, we must divide the range of values by two and take into account a suitable offset for the zero point.

Sound pressure level

In the field of acoustic measurements, the sound pressure level L_p is measured in decibels. L_p is the logarithmic ratio of sound pressure p referred to a sound pressure p_0 = 20 µPa (micro pascals). Sound pressure p_0 is the lower limit of the pressure which the human ear can perceive in its most sensitive frequency range (around 3 kHz). This pressure level is known as the threshold of hearing.

$$L_p = 20 \cdot \lg\left(\frac{p}{p_0}\right) \mathrm{dB}$$

$$p = 10^{\frac{L_p}{20}} \cdot p_0$$

Weighted sound pressure level dB(A)

The human ear has a rather pronounced frequency response which also depends on the sound pressure level. When measuring sound pressure, weighting filters are used to simulate this frequency response. This provides us with level values which come closer to simulating human loudness perception compared to unweighted levels. The different types of weighting filters are known as A, B, C and D.

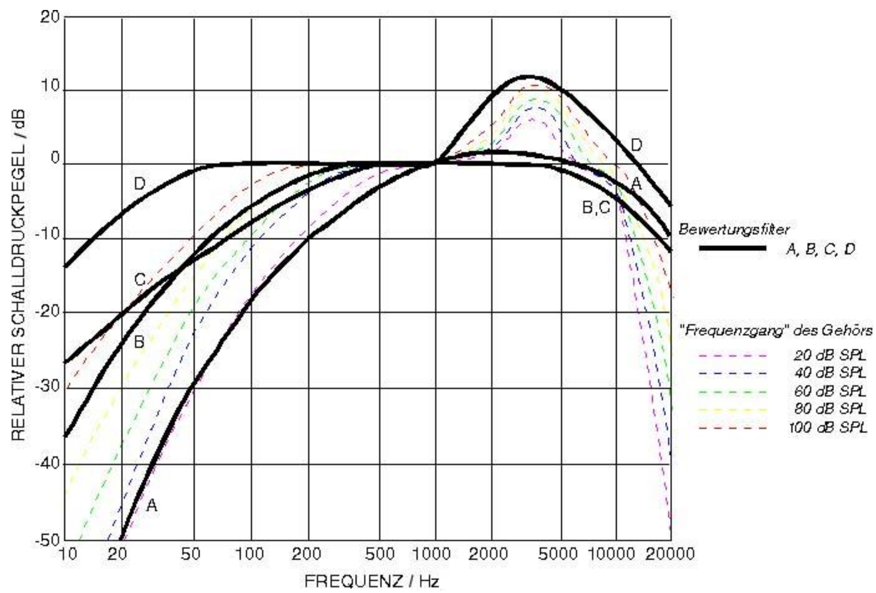

Fig 16: Weighting filters A, B, C and D and the frequency response of human hearing

The A filter is used the most. The level measured in this manner is known as L_{pA} and is specified in units of **dB(A)** to designate the weighting filter.

A difference in sound pressure level of 10 dB(A) is perceived as roughly a doubling of the volume. Differences of 3 dB(A) are clearly audible. Smaller differences in sound level can usually be recognized only through direct comparison.

Examples: *Our hearing range extends from 0 dB(A) (threshold of hearing) up to the threshold of pain at about 120 dB(A) to 134 dB(A). The sound pressure level in a very quiet room is approximately 20 dB(A) to 30 dB(A). Using 16 data bits, the dynamic range of a music CD can reach 98 dB, sufficient to satisfy the dynamic range of the human ear.*

11 A few numbers worth knowing

Working with decibel values is a lot easier if you memorize a few key values. From just a few simple values, you can easily derive other values when needed. We can further simplify the problem by rounding exact values up or down to some easy to remember numbers. All we have to do is remember the simplified values, e.g. a power ratio of 2 corresponds to 3 dB (instead of the exact value of 3.02 dB which is rarely needed).

The following table lists some of the most useful numbers to remember.

Table for conversion between decibels and linear values

	Power ratio		**Voltage ratio**	
dB value	Rough	Exact	Rough	Exact
0.1 dB	**±2 %**	+2.3 % -2.3%	**±1 %**	+1.16 % -1.15 %
0.2 dB	**±4 %**	+4.7 % -4.5 %	**±2%**	+2.33 % -2.23 %
0.5 dB	**±10 %**	+12.2 % -10.9 %	**±5 %**	+5.9 % -5.5 %
1 dB	**± 20 %**	+25.9 % -20.5 %	**±10 %**	+12.2 % -11.9 %
3 dB	**2** **0.5**	1.995 0.501	**1.4** **0.7**	1.412 0.798
3.02 dB	**2** **0.5**	2.0 0.5	**1.414** **0.707**	$\sqrt{2}$ $1/\sqrt{2}$
5 dB	**3** **0.33**	3.16 0.316	**1.8** **0.6**	1.778 0.562
6 dB	**4** **0.25**	3.98 0.25	**2** **0.5**	1.995 0.501
10 dB	**10** **0.1**	10 0.1	**3** **0.3**	3.162 0.316
20 dB	**100** **0.01**	100 0.01	**10** **0.1**	10 0.1
40 dB	**10000** **0.0001**	10000 0.0001	**100** **0.01**	100 0.01
60 dB	**1000000** **0.000001**	1000000 0.000001	**1000** **0.001**	1000 0.001

Table 3: Conversion between decibels and linear values

From this table, you should probably know at least the rough values for 3 dB, 6 dB, 10 dB and 20 dB by heart.

***Note**:* *3 dB is not an exact power ratio of 2 and 6 dB is not exactly 4! For everyday usage, however, these simplifications provide sufficient accuracy and as such are commonly used.*

Intermediate values which are not found in the table can often be derived easily:

4 dB = 3 dB + 1 dB, corresponding to a factor of 2 + 20% of the power, i.e. approx. 2.4 times the power.

7 dB = 10 dB – 3 dB, corresponding to 10 times the power and then half, i.e. 5 times the power.

Table for adding decibel values

If you need to compute the sum of two values specified in decibels precisely, you must convert them to linear form, add them and then convert them back to logarithmic form. However, the following table is useful for quick calculations. Column 1 specifies under Delta dB the difference between the two dB values. Column 2 specifies a correction factor for power quantities. Column 3 specifies a correction factor for voltage quantities. Add the correction factor to the higher of the two dB values to obtain the total.

Delta dB	Power	Voltage
0	3.01	6.02
1	2.54	5.53
2	2.12	5.08
3	1.76	4.65
4	1.46	4.25
5	1.19	3.88
6	0.97	3.53
7	0.79	3.21
8	0.64	2.91
9	0.51	2.64
10	0.41	2.39
11	0.33	2.16
12	0.27	1.95
13	0.21	1.75
14	0.17	1.58
15	0.14	1.42
16	0.11	1.28
17	0.09	1.15
18	0.07	1.03
19	0.05	0.92
20	0.04	0.83

Table 4: Correction factors for adding decibel values

Examples: *1) Suppose we would like to add power levels of -60 dBm and -66 dBm. We subtract the decibel values to obtain a difference of 6 dB. From the table, we read off a correction factor of 0.97 dB. We add this value to the higher of the two values, i.e. -60 dBm (-60 dBm is greater than -65 dBm!) and obtain a total power of -59 dBm.*

2) When we switch on a signal, the noise displayed by a spectrum analyzer increases by 0.04 dB. From the table, we can see that the level of this signal lies about 20 dB below the noise level of the spectrum analyzer.

3) We would like to add two equal voltages. This means that the level difference is 0 dB. The total voltage lies 6 dB (value from the table) above the value of one voltage (= twice the voltage).

Some more useful values

The following values are also useful under many circumstances:

13 dBm corresponds to U_{RMS} = 1 V into 50 Ω

0 dBm corresponds to U_{RMS} = 0.224 V into 50 Ω

107 dB(μV) corresponds to 0 dBm into 50 Ω

120 dB(μV) corresponds to 1 V

-174 dBm is the thermal noise power in 1 Hz bandwidth at a temperature of approx. 20 °C (68 °F).

Other reference quantities

So far, we have used 1 mW and 50 Ω as our reference quantities. However, there are other reference systems, including most importantly the 75 Ω system in television engineering and the 600 Ω system in acoustic measurement technology. The 60 Ω system formerly used in RF technology and the 600 Ω system in the United States with a reference value of 1.66 mW are now rather rare. However, it is easy to adapt the formulas given above to these reference systems.

R	P_0	U_0	Note
50 Ω	1 mW	0.224 V	RF engineering
60 Ω	1 mW	0.245 V	RF engineering (old)
75 Ω	1 mW	0.274 V	TV engineering
600 Ω	1 mW	0.775 V	Acoustics
600 Ω	1.66 mW	1.000 V	US standard

Table 5: Additional reference systems

Accuracy, number of decimal places

How many decimal places should be used when we specify decibel values?

If we increase a value *x* which is a power quantity specified in decibels by 0.01 dB, the related linear value will change as follows:

$$x\,\mathrm{dB} + 0.01\,\mathrm{dB} \equiv 10^{\frac{x+0,01}{10}} = 10^{\frac{x}{10}} \cdot 10^{\frac{0,01}{10}} = 10^{\frac{x}{10}} \cdot 1.0023$$

This is equivalent to a 0.23% change in the power. Voltage quantities change by only 0.11%. These minor changes cannot be distinguished from normal fluctuations of the measurement result.

Accordingly, it does not make sense to specify decibel values with, say, five or more decimal places, except in a few rare cases.

12 Bibliography

[1] Field Strength and Power Estimator, Application Note 1MA85, Rohde & Schwarz GmbH &Co. KG **1MA85**

[2] For further explanation of the terminology used in this Application Note, see also **www.wikipedia.org**.

[3] Christoph Rauscher, Fundamentals of Spectrum Analysis, Rohde & Schwarz GmbH&Co. KG, PW 0002.6629.00

13 Additional information

We welcome your comments and questions relating to this Application Note. You can send them by e-mail to: **TM-Applications@rsd.rohde-schwarz.com**.

Please visit the Rohde & Schwarz website at **www.rohde-schwarz.com**. There, you will find additional Application Notes and related information.

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